The effects of static contact angles on standing waves

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Damping rates and natural frequencies of the fundamental axisymmetric mode in circular cylinders were measured when the contact angle between the water and the side walls was acute, obtuse, and about $\pi/2$. Damping rates decreased with increasing contact angle; natural frequencies increased with increasing contact angle.

The classical, linear, boundary-value problem for inviscid, gravity–capillary waves in closed basins with vertical side walls neglects capillarity at the contact line, which interfaces the air, liquid, and side wall. Thus it assumes that the contact angle, θ, between the liquid and the side walls is $\pi/2$, both with and without waves (Lamb[1]). (The contact angle is the angle between a line that is tangent to the meniscus at the contact line and the vertical side wall, such that θ is acute for a hydrophillic liquid and obtuse for a hydrophobic liquid.) Similarly, classical calculations that use a laminar boundary-layer analysis to predict the effects of weak viscosity in the liquid also assume $\theta=\pi/2$ (Case and Parkinson[2]). Recent theoretical work (e.g., Hocking[3] and Miles[4,5]) incorporates capillarity into the boundary condition at the contact line, but generally neglects the mensicus; i.e., it also requires $\theta=\pi/2$. Miles[5] states that the error associated with neglecting the static contact angle is $O(\kappa)$, in which $\kappa=kl\ll 1$, k is the wave number obtained from the classical analysis, $l = \sqrt{T/\rho g}$ is the capillary length ($=0.27$ cm for a pure, air–water interface), T is the surface tension of the air–liquid interface, ρ is the liquid density, and g is the acceleration of gravity.

In this Brief Communication we report results of experiments using water–basin combinations for which the static contact angle was less than, greater than, and about equal to $\pi/2$. The purpose of the experiments is to determine the qualitative behavior of damping rates and natural frequencies in consequence of acute and obtuse static menisci. To this end, we measured natural frequencies and damping rates of the (0,1) mode (the standing wave with zero nodal diameters and one nodal circle) at an air–water interface in four circular cylinders with radii $R=2.77$ cm (so that $\kappa\cong 0.3$). The cylinders were constructed of annealed glass ($\theta\cong\pi/2$), polystyrene ($\theta<\pi/2$), acrylic ($\theta<\pi/2$), and Teflon ($\theta>\pi/2$). Experiments were conducted on a pure air–water interface and on one saturated with an insoluble, monomolecular film of diolein.

We have no information on dynamical aspects of contact lines and angles during wave passage. However, Cocciaro *et al.*[6] found experimentally that the dynamical behavior of the contact angle depends on the speed of the contact line. When this speed exceeds a critical value the contact angles of the advancing and receding liquid have almost constant (but not necessarily the same) values, and the classical boundary-value problem is expected to apply. Ablett[7] measured a critical speed of 0.4 mm/sec. The steady-state wave amplitudes in our experiments were about 5 mm and the frequencies were about 6 Hz, so contact-line speeds were about 100 mm/sec; hence the classical theory is expected to apply.

The surface displacement obtained from the classical, linear boundary-value analysis of unforced fluid oscillations for the (0,1) mode in a circular cylinder is

$$\eta(r,t)=A_{01}\cos\omega_{01}tJ_0(k_{01}r), \tag{1}$$

where A_{01} is the wave amplitude, ω_{01} is the (radian) natural frequency, J_0 is the zeroth-order Bessel function, and k_{01} is the wave number found from $J_0'(k_{01}R)=0$. The natural frequency and wave number are related by the dispersion relation

$$\omega_{01}=[(gk_{01}+Tk_{01}^3/\rho)\tanh k_{01}h]^{1/2} \tag{2}$$

in which h is the fluid depth. The natural frequency decreases a few percent in consequence of wave damping that arises from laminar boundary layers around the wetted surfaces. When the wave is excited by vertical oscillations of the cylinder at a frequency 2ω, the wave frequency is not the natural frequency, but half the forcing frequency. When the forcing is stopped, the wave amplitude decays with an exponential rate γ, and the frequency shifts a small amount Δ, so that during wave damping, the water surface displacement is given by

$$\eta(r,t)=A_{01}e^{-\gamma t}\cos(\omega_{01}-\Delta)tJ_0(k_{01}r). \tag{3}$$

The frequency shift occurs because of the difference between the natural and forced wave frequencies and is equal to

$$\Delta=(\omega^2-\omega_{01}^2)/2\omega. \tag{4}$$

The linear, boundary-layer analysis predicts that the damping rate is $\gamma_p=0.12$ rad/sec, when the air–water interface is pure with $T=72.4$ dyn/cm. Miles[8] calculated the

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TABLE I. Measured damping rates and natural frequencies of the (0,1) mode in four cylinders when the air–water interface was pure and when covered with a saturated film of diolein. Predicted values of damping rate are $\gamma_{\text{pure}}=0.12$ rad/sec and $\gamma_{\text{dio}}=0.68$ rad/sec. Predicted values of natural frequency are $f_{\text{pure}}=6.23$ Hz and $f_{\text{dio}}=6.08$ Hz.

θ (rad)/ material	γ_{pure} (rad/sec)	γ_{dio} (rad/sec)	f_{pure} (Hz)	f_{dio} (Hz)
$\pi/2$ glass	0.14	0.68	6.62	6.42
$>\pi/2$ Teflon	0.14	0.58	6.76	6.56
$<\pi/2$ acrylic	0.18	0.90	6.49	6.39
$<\pi/2$ polystyrene	0.19	0.80	6.49	6.33

contribution to the damping rate due to an elastic film at the surface. This contribution depends on the change of surface tension with the film's concentration. Henderson *et al.*[9] measured damping rates of the (0,1) mode in the annealed glass cylinder due to insoluble films of diolein. They also measured the dependence of surface tension on film concentration. When they used these measurements in Miles theory, they calculated a theoretical damping rate for a saturated diolein film of 0.68 rad/sec.

The experimental apparatus is described in detail by Henderson *et al.*[10] In brief, waves were excited by vertical oscillations of the cylinder of fluid with a servo-controlled electromagnetic motor. The water was doubly distilled and filtered of particles larger than 11 μm. All of the cylinders were cleaned by rinses in sodium decyl sulphate and distilled water. (This cleaning agent was used since results for the glass cylinder were the same using it and using chromic acid.) The water surface was vacuumed of impurities with a micropipette until the water depth was 2.00 cm. Surface tension was measured with a Wilhelmy plate connected to a Statham transducer. The cylinder was oscillated vertically at a frequency of 12.934 60 Hz. The (0,1) mode with forced frequency 6.467 30 Hz grew and obtained a steady-state amplitude. The forcing was stopped and a time series of the surface displacement was measured with an *in situ*, capacitance-type probe that was located in the center of the water surface. The time series was low-passed filtered at 30 Hz, digitized at 350 Hz, and complex-demodulated at the forced-wave frequency. The complex demodulating procedure provides both phase and amplitude information; graphs of the amplitude envelope provided the damping rate γ, and graphs of the phase provided the frequency shift Δ. We determined the natural frequency from the measured value of Δ and (4). Next, 5 μl of diolein solution was spread on the water surface creating a measured surface tension of 46.6 dyn/cm. The damping rate and natural frequency of the wave on this saturated surface were measured. This procedure was repeated for each of the four cylinders.

The results of our (large-amplitude) experiments are reported in Table I. When the surface was pure, the measured damping rates were about 1.2 times larger than predicted for $\theta \geqslant \pi/2$ and about 1.5 times larger than predicted for $\theta<\pi/2$. When the surface was covered with a saturated film of diolein the measured damping rates were 0.85 times the predicted value for $\theta>\pi/2$, they were in agreement with predictions for $\theta \cong \pi/2$, and they were 1.2 times larger than predictions for $\theta<\pi/2$. The measured natural frequency was 1.08 times larger when $\theta>\pi/2$, it was about 1.06 times larger than predicted when $\theta \cong \pi/2$, and it was about 1.04 times larger than predicted when $\theta<\pi/2$.

Similar experiments for $\theta=0$ were conducted by Henderson and Miles[11] and Cocciaro *et al.*[6] Henderson and Miles[11] measured the damping rates of the (large-amplitude) (0,1) mode in a circular cylinder. Their measurements were about 1.6 times larger than predicted. They used water with a wetting agent (Photo Flo 200) added. This detergent created a film on the side walls, so that the contact line rose and fell on the film, rather than on the glass walls and the contact angle was $\theta=0$. The wetting agent also created a saturated film at the air–water interface. They had no information on the elastic properties of this film, but could not consider the surface to be pure; thus, they compared their results to the (classical) damping model for an inextensible surface. They also measured natural frequencies and found them to be in reasonable agreement with predictions that were corrected by laminar boundary-layer damping.

Cocciaro *et al.*[6] measured the damping rates of the (small-amplitude) (1,0) mode in a circular cylinder for which $\theta=0$. Their measurements were also about 1.6 times larger then predicted by the classical theory when wave amplitudes were larger than a critical value that corresponded to a critical contact-line speed. Below that critical value the damping rates increased with decreasing amplitude. They hypothesized that this amplitude dependence resulted from contact-line motion, which becomes increasingly important as the wave amplitude (and correspondingly the speed of the contact line) becomes small. They also measured natural frequencies and found them to be in reasonable agreement with predictions.

Our measurements of damping rates were minimum for obtuse contact angles and maximum for acute contact angles. In addition, the damping rates for acute (but nonzero) angles were better predicted than those measured by Henderson and Miles[11] and Cocciaro *et al.*[6] who conducted similar experiments with $\theta=0$. Thus, we note that damping rates decrease with increasing contact angle.

Our measured natural frequencies were much larger than predicted, even when $\theta \cong \pi/2$. The maximum discrepancy occurred when the contact angle was obtuse. The minimum discrepancy occurred when the contact angle was acute. Henderson and Miles[11] and Cocciaro *et al.*[6] found reasonable agreement with $\theta=0$. Thus, we note that natural frequencies increase with increasing contact angle.

We conclude that it is best to use fluid–basin combinations that give rise to obtuse contact angles in order to minimize wave damping rates. We conclude that it is best to use fluid–basin combinations that give rise to zero contact angles to minimize differences between predicted and measured natural frequencies.

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