Constant-Phase-Element Behavior Caused by Resistivity Distributions in Films

II. Applications

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A normal power-law distribution of local resistivity with a uniform dielectric constant was found to be consistent with the constant-phase element (CPE). An analytic expression, based on the power-law resistivity distribution, was found that relates CPE parameters to the physical properties of a film. This expression worked well for such diverse systems as aluminum oxides, oxides on stainless steel, and human skin. Good values for film thickness were obtained, even when previously proposed expressions could not be used or yielded incorrect results. The power-law model yields a CPE impedance behavior in an appropriate frequency range, defined by two characteristic frequencies. Ideal capacitive behavior is seen above the upper characteristic frequency and below the lower characteristic frequency. A symmetric CPE response between the characteristic frequencies can be obtained by adding a parallel resistive pathway.

Normal distributions of time-constants can be expected in systems such as kinetics, mass transfer, and reaction mechanisms, as well as material properties, such as permittivity and conductivity. The constant-phase element (CPE) model, which is purely a mathematical description, may adequately represent impedance data, but it gives no insight into the physical processes that yield such a response. However, capacitance values are often extracted from CPE data using such expressions as

\[ C = Q^{1/a} \left( R_e^{1-a} + R_i^{(1-a)/a} \right) \]

developed by Brug et al.1 or

\[ C_{HM} = Q^{1/a} R_i^{(1-a)/a} \]

presented by Hsu and Mansfeld.2 The parameters \( R_e \) and \( R_i \) in Eq. 1 represent the Ohmic and charge-transfer resistances, respectively, and the parameter \( R_i \) in Eq. 2 represents the film resistance. The subscripts B and HM refer to the authors of the respective papers. Often, the capacitance values obtained are used to estimate the thickness of dielectric layers. Recently, Hirschorn et al.3 associated these expressions unambiguously with either surface or normal time-constant distributions and demonstrated the importance of using the correct formula that corresponds to a given type of distribution. Limitations of Eq. 2 for predicting layer thickness were discussed. For a blocking system, where the film resistance is infinite, Eq. 2 cannot be used as it would yield an infinite capacitance. When the distribution of physical properties is broad, Eq. 2 may yield an inaccurate estimate of capacitance. Their work demonstrated the importance of developing physically reasonable models that can result in CPE behavior.

Normal distributions of time-constants can be expected in systems such as oxide films, organic coatings, and human skin. Such normal time-constant distributions may be caused by distributions of resistivity and/or dielectric constant. Hirschorn et al.4 used a power-law distribution of resistivity

\[ \frac{\rho}{\rho_0} = \left( \frac{\rho_0}{\rho} \right) \left( 1 - \frac{\rho_0}{\rho} \right)^{g-1} \]

where the parameters \( \rho_0 \) and \( \rho_0 \) are the boundary values of resistivity at the interfaces, to show that, for \( \omega < (\rho_0 \varepsilon \varepsilon_0)^{-1} \), an analytic expression for the impedance could be obtained as

\[ Z_f(\omega) = g \frac{\rho_0^{(1-\alpha)}}{(\rho_0 + j\omega \varepsilon \varepsilon_0)^{\alpha}} \]

where

\[ \alpha = \frac{\gamma - 1}{\gamma} \]

The function \( g \) was evaluated numerically and could be expressed as

\[ g = 1 + 2.88(1 - \alpha)^{2.75} \]

This work showed that under assumption that the dielectric constant is independent of position, a normal power-law distribution of local resistivity can be consistent with the CPE.4

The objective of the present work is to develop a method to extract physically meaningful parameters from impedance data yielding CPE behavior corresponding to systems for which a variation of properties is expected in the direction normal to the electrode. The resulting approach is applied to experimental data. The data reexamined here were already published, and the experimental details may be found in the references.

Method

In many cases, the impedance response follows CPE behavior, expressed for blocking systems as

\[ Z_{CPE} = \frac{1}{(j\omega)^{\alpha}Q} \]

and for reactive systems as

\[ Z_{CPE} = \frac{1}{(j\omega)^{\alpha}Q} \]
with the impedance obtained by numerical integration of Eq. 4 is consistent with the CPE is presented in Fig. 1 for Equation 9 provides a new means of relating CPE parameters to the physical properties of the film.

The frequency range for which the impedance response provided by Eq. 4 is consistent with the CPE is presented in Fig. 1 for $\gamma = 4$ ($\alpha = 0.75$). The symbols represent the product of $Q$, from Eq. 9, with the impedance obtained by numerical integration of

$$Z_{\text{CPE},R_f} = \frac{R_t}{1 + (j\omega)^{\alpha}QR_t}$$  \[8\]

where $\alpha$ and $Q$ are constants, independent of frequency. For $(\rho_a\varepsilon_0)^{-1} < \omega < (\rho_b\varepsilon_0)^{-1}$, Eq. 4 takes the form of Eq. 7 with

$$Q = \frac{(\varepsilon_0\varepsilon_a)^{\alpha}}{\bar{\gamma} \rho_b^{\alpha}}$$  \[9\]

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$$Z_0(\omega) = \int_0^{b} \frac{\rho(x)}{1 + j\omega\varepsilon_0 \rho(x)} dx$$  \[10\]

where the resistivity is given by Eq. 3. The line represents $(j\omega)^{-0.75}$ in agreement with Eq. 7, and the characteristic frequencies are defined in terms of the parameters associated with Eq. 3 as $f_b = (2\pi\rho_b\varepsilon_b)^{-1}$ and $f_0 = (2\pi\rho_0\varepsilon_0)^{-1}$.

The impedance behaviors near frequencies $f_0$ and $f_b$ are presented in Nyquist format in Fig. 2a and b, respectively. The response is capacitive at frequencies lower than $f_0$ and at frequencies higher than $f_b$. The impedance response reflects the distributed resistivity at frequencies between $f_0$ and $f_b$.

Even if $\varepsilon$ is known from independent measurements, operation in the frequency range $f_b < f < f_b$ provides only the product $\delta\rho_b^{1-\alpha}$ given by Eq. 9. Therefore, measurements for $f > f_b$ are required to obtain separately the film thickness $\delta$ and the interfacial resistivity $\rho_b$. Measurements for $f < f_b$ are required to obtain the interfacial resistivity $\rho_0$. The maximum experimental frequency range for electrochemical systems is generally considered to be of the order of 1 mHz–100 kHz. The impedance response given in Fig. 1 is presented for a larger frequency range to explore the consequence of the large variation possible in the resistivity parameters $\rho_0$ and $\rho_b$.

Results and Discussion

As discussed above, the characteristic frequencies $f_0$ and $f_b$ define the range for which the system behaves as a CPE. In addition, the characteristic frequency $f_0$ represents the frequency at the maximum magnitude of the imaginary part of the impedance. Since the frequencies $f_0$ and $f_b$ depend on the film properties $\varepsilon$, $\rho_0$, and $\rho_b$, the subsequent discussion is organized according to the respective values of $\rho_0$ and $\rho_b$.

Aluminum oxide (large $\rho_0$ and small $\rho_b$).—When the resistivity limit $\rho_b$ is small, the high-frequency domain corresponds to a CPE. When the resistivity limit $\rho_0$ is very large, its influence on the impedance response can be outside the experimentally accessible frequency range. In this case, the resulting impedance response is that of a blocking electrode, as is seen in Fig. 3a for a frequency range that excludes $f_0$ and $f_b$. Such a behavior can be observed experimentally, as is shown in Fig. 3b for passive aluminum in a 0.1 M Na$_2$SO$_4$ electrolyte. The dashed line in Fig. 3b represents a CPE fit to the data according to Eq. 7.

The effective capacitance of a film can be expressed as

$$C_{\text{eff}} = \frac{\varepsilon_0\varepsilon_a}{\delta}$$  \[11\]

Introduction of Eq. 11 into Eq. 9 yields

$$f_0 = \frac{2\pi\rho_0\varepsilon_0}{1 - j\omega\varepsilon_0 \delta}$$  \[12\]

$$f_b = \frac{2\pi\rho_b\varepsilon_b}{1 - j\omega\varepsilon_0 \delta}$$  \[13\]

Figure 1. Representation of $ZQ$ where $Z$ is generated by numerical integration of Eq. 10 and $Q$ is obtained from Eq. 9 for $\gamma = 4$ ($\alpha = 0.75$) and $\varepsilon = 10$ with $\rho_0$ and $\rho_b$ as parameters: (a) the real component of impedance and (b) the imaginary component of impedance. The line represents $(j\omega)^{-0.75}$ in agreement with Eq. 7. The symbols represent calculations performed for $\Delta\rho_0 = \rho_0 = 10^{15}$ Ω cm and $\rho_b = 10^{-1}$ Ω cm; (1) $\rho_0 = 10^{14}$ Ω cm; $\rho_b = 10^{2}$ Ω cm; and (2) $\rho_0 = 10^{10}$ Ω cm; $\rho_b = 10^{5}$ Ω cm. The characteristic frequencies are $f_b = (2\pi\rho_b\varepsilon_b)^{-1}$ and $f_0 = (2\pi\rho_0\varepsilon_0)^{-1}$.

Figure 2. Nyquist plot of the data presented in Fig. 1 for $\rho_0 = 10^{10}$ Ω cm and $\rho_b = 10^{1}$ Ω cm: (a) plot showing the characteristic frequency $f_b = (2\pi\rho_b\varepsilon_b)^{-1}$ = 18 Hz and (b) zoomed region showing the characteristic frequency $f_0 = (2\pi\rho_0\varepsilon_0)^{-1} = 1.8 \times 10^6$ Hz.
represents the capacitance of the layer. The parameters Eq. 12 can be obtained from graphical analysis of impedance data,6 according to the literature, the dielectric constant varies between 7.5 and 15.7 Under assumption of a mean value of 11.5 for \( \epsilon \), a minimum film thickness can be estimated to be 9 nm. Following Eq. 13, \( \rho_{b,\text{max}} = 5.2 \times 10^8 \) \( \Omega \) cm. In a fashion similar to the development of \( \rho_{b,\text{max}} \), a minimum value \( \rho_{b,\text{min}} \) can be obtained from the lowest measured frequency \( f_{\text{min}} = 0.1 \) Hz to be \( 1.6 \times 10^{12} \) \( \Omega \) cm. Therefore, the minimum amplitude of resistivity variation within the alumina layer is \( 5.2 \times 10^6 \)–\( 1.6 \times 10^{12} \) \( \Omega \) cm. These values fall within the ranges typical of semiconductors and insulators, respectively.

The application of the Hsu–Mansfeld formula, given as Eq.12, is not possible because a capacitive loop is not apparent in the experimental results and a value for \( R_L \) cannot be estimated. This example shows that application of a power-law distribution of resistivity allows estimation of film thickness of a blocking film for which the existing formulas do not apply. In the present case, the lower limit for the thickness and the minimum range of resistivity values are determined.

Stainless steel (finite \( \rho_b \) and small \( \rho_d \)) — When \( \rho_b \) is sufficiently small, the characteristic frequency \( f_b \) falls within the experimental frequency range. Under these conditions, a finite value is obtained for the impedance at low frequencies.

An experimental example can be given with the impedance of oxides developed on stainless steel. Experimental data are shown in Fig. 6a for the impedance response of a Fe17Cr disk for the passive domain for 1 h at −0.1 V measured with respect to a mercury/mercurous sulfate electrode in saturated K2SO4 electrolyte (data taken from Jorcin et al.). The dashed line represents a CPE fit to the data according to Eq.7.

\[
C_{\text{eff}} = Q(\rho_b \epsilon \Delta \alpha)^{-1-g} \tag{12}
\]

When \( \alpha = 1, g = 1 \), and, as shown by Eq. 12, the CPE parameter \( Q \) represents the capacitance of the layer. The parameters \( Q \) and \( \alpha \) in Eq. 12 can be obtained from graphical analysis of impedance data,6 and \( \epsilon \) is often known from independent measurements. The parameter \( \rho_b \) cannot be known exactly for data showing high-frequency CPE behavior. Nevertheless, Eq. 12 may be attractive for analysis of CPE data for which a polarization resistance cannot be estimated due to blocking behavior in the measured frequency range or for which the polarization resistance is influenced by phenomena not associated with the dielectric response of the film.

Although the value of \( \rho_b \) is unknown for data showing high-frequency CPE behavior, an upper bound on its value can be defined because the characteristic frequency \( f_b \) must be larger than the largest measured frequency \( f_{\text{max}} \). Thus, a maximum value of \( \rho_b \) can be obtained

\[
\rho_{b,\text{max}} = \frac{1}{2\pi\epsilon\Delta \alpha f_{\text{max}}} \tag{13}
\]

The relationship between \( \rho_{b,\text{max}} \) and \( f_{\text{max}} \) for different values of dielectric constant is shown in Fig. 4. Since Eq. 12 can be written as

\[
C_{\text{eff}} = Q(2\pi f_b)^{-1-g} \tag{14}
\]

a similar bound on the effective capacitance can be found to be

\[
C_{\text{max}} = Q(2\pi f_{\text{max}})^{-1-g} \tag{15}
\]

Therefore

\[
C_{\text{eff}} \frac{C_{\text{max}}}{} = \left( \frac{f_b}{f_{\text{max}}} \right)^{-1-g} \left( \frac{\rho_{b,\text{max}}}{\rho_b} \right)^{-1} \tag{16}
\]

The uncertainty in calculating effective capacitance due to uncertainty in the value of \( \rho_b \) can be ascertained from the results presented in Fig. 5. When \( \alpha \) is close to unity, the estimation of \( C_{\text{eff}} \) from Eq. 12 is relatively insensitive to the value of \( \rho_b \), whereas, for \( \alpha = 0.5 \), an uncertainty of 2 orders of magnitude in \( \rho_b \) results in an uncertainty of 1 order of magnitude in \( C_{\text{eff}} \).

In Fig. 3b, the impedance of a passive aluminum electrode was given, where the highest measured frequency \( f_{\text{max}} \) was 30 kHz. The corresponding CPE parameters were \( \alpha = 0.77 \) and \( Q = 1.7 \times 10^{-3} \) F cm\(^{-2}\) s\(^{-0.23}\). Following Eq. 15, \( C_{\text{max}} = 1.1 \) \( \mu \)F/cm\(^2\). According to the literature, the dielectric constant varies between 7.5 and 15.7 Under assumption of a mean value of 11.5 for \( \epsilon \), a minimum film thickness can be estimated to be 9 nm. Following Eq. 13, \( \rho_{b,\text{max}} = 5.2 \times 10^8 \) \( \Omega \) cm. In a fashion similar to the development of \( \rho_{b,\text{max}} \), a minimum value \( \rho_{b,\text{min}} \) can be obtained from the lowest measured frequency \( f_{\text{min}} = 0.1 \) Hz to be \( 1.6 \times 10^{12} \) \( \Omega \) cm. Therefore, the minimum amplitude of resistivity variation within the alumina layer is \( 5.2 \times 10^6 \)–\( 1.6 \times 10^{12} \) \( \Omega \) cm. These values fall within the ranges typical of semiconductors and insulators, respectively.

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The film thickness results. This value of Eq. 3 provided a calculated response that agrees with the experimental

$\text{Cr} \equiv 4.5 \times 10^{-5} \text{ F cm}^{-2} \text{s}^{-0.11}$; and $b = 1.2-12.6 \text{ nm}$, which encompasses the value of 3 nm obtained from XPS. For $\alpha = 0.89$, an uncertainty in $\rho_b$ of 9 orders of magnitude yielded an uncertainty in $\delta$ of only 1 order of magnitude. In many cases, a tighter bound on the estimated $\rho_b$ may be possible, as is the case for human stratum corneum presented in a subsequent section.

In contrast to the data presented for aluminum oxide, a capacitive loop is evident in the impedance response for oxides on steel. The zero-frequency impedance was estimated using an extension of the measurement model approach$^1$ to be $R_i = 0.756 \pm 0.09 \text{ M}\Omega \text{ cm}^2$. The extrapolation is shown in Fig. 6b. The corresponding estimate of film thickness from the Hsu–Mansfeld formula, given as Eq. 2, is $\delta = 0.190 \text{ nm}$. A similar extrapolation to zero frequency, also shown in Fig. 6b, was performed using the power-law model, yielding $R_i = 0.85 \text{ M}\Omega \text{ cm}^2$. The corresponding estimate of film thickness from the Hsu–Mansfeld formula is $\delta = 0.187 \text{ nm}$. Independent of the method used to extrapolate the impedance to zero frequency, the film thickness value obtained from application of the Hsu–Mansfeld formula is substantially smaller than the experimentally measured value of 3 nm. In contrast, application of a power-law distribution of resistivity provides an estimation of film thickness that encompasses the experimentally determined value.

**Human skin (power law with parallel path).**—Impedance data from Membrino$^6$ are presented in Fig. 8 for heat-separated excised human stratum corneum immersed in 50 mM buffered CaCl$_2$ electrolyte for 1.9 h. An analysis based on application of Eq. 2 was presented by Hirshorn et al.,$^7$ who, using graphical methods,$^8$ found that $\alpha = 0.834$ and $Q = 5.36 \times 10^{-5} \text{ F cm}^{-2} \text{s}^{-0.166}$. They reported that the skin thickness estimated using the capacitance from Eq. 2 and a dielectric constant of 49 was 2.6 $\mu\text{m}$, about 1 order of magnitude smaller than the expected value.

Application of Eq. 9 for estimation of film thickness requires an estimate for $\rho_b$. As seen by the straight line in Fig. 8c, CPE behavior is evident at the largest measured frequencies. Thus, an upper bound for $\rho_b$ can be established from the maximum measured frequency of 21 kHz to be $1.7 \times 10^{0} \text{ F cm}^{-2}$. The resistivity of body fluids is $48 \text{ F cm}$. If this value is considered to be a lower bound, the range of possible $\rho_b$ values is $1.7 \times 10^{6}-48 \text{ F cm}^{-2}$. For $\alpha = 0.834$, $g = 1.04$ from Eq. 6.

The corresponding estimated thickness of skin is between 6 and 31 $\mu\text{m}$, in good agreement with the expected value of 10–40 $\mu\text{m}$. The thickness estimated using Eq. 12 is in better agreement with expected values than are the values obtained by use of Eq. 2. Hirshorn et al.$^7$ explained that the capacitance obtained using Eq. 2 does not account properly for the low resistivity regions of skin that have characteristic frequencies outside the measured frequency range. Equation 2 is based on calculation of the characteristic $RC$ time constant and does not take any specific distribution of resistivity or dielectric constant into account. The better agreement obtained using Eq. 12 can be explained by the fact that it is based on formal

![Figure 6. Impedance diagram of oxide on a Fe17Cr stainless steel disk (symbols): (a) experimental frequency range. The solid line is the power-law model following Eq. 4 with parameters $\rho_b = 4.5 \times 10^{13} \text{ F cm}^{-2} \text{s}^{-0.11}$ and $Q = 3.7 \times 10^{-6} \text{ F cm}^{-2} \text{s}^{-0.11}$; and (b) extrapolation to zero frequency where the dashed line represents the fit of a Voigt measurement model and the solid line represents the fit of the power-law model.](image1)

![Figure 7. Impedance response of oxide on a Fe17Cr stainless steel disk (symbols) and the theoretical model (line) with parameters reported in Fig. 6a: (a) the real component and (b) the imaginary component. The electrolyte resistance value was 23 $\Omega \text{ cm}^2$.](image2)
solution for the impedance associated with a specified resistivity distribution and requires only the high-frequency portion of the measurement.

The power-law impedance model may also be applied to explore the low-frequency impedance response for skin. The low-frequency limit for the impedance response expressed as Eq. 4 is

$$Z_0(0) = g \rho_0^{1-n} \rho_0^n$$  \hspace{1cm} (17)

The function $g$ can be eliminated by introduction of Eq. 9 to yield

$$Z_0(0) = \frac{(\rho_0 \delta \varepsilon_0)^n}{Q}$$  \hspace{1cm} (18)

The value of $\rho_0 = 2.2 \times 10^8 \Omega \cdot \text{cm}$ can be obtained from the characteristic frequency $f_0 = 170 \text{ Hz}$ by using the relationship

$$\rho_0 = \frac{1}{2 \pi \varepsilon_0 f_0}$$  \hspace{1cm} (19)

and the maximum value of the impedance is obtained from Eq. 18 to be $Z_f(0) = 56 \text{ k}\Omega \cdot \text{cm}^2$. Under the assumption that $\rho_0 = 48 \text{ k}\Omega \cdot \text{cm}$ (or $\delta = 31 \mu\text{m}$), Eq. 4 yields the impedance simulation shown as a dashed line in Fig. 8. As discussed in a companion paper, the impedance response is asymmetric in a Nyquist plot, yielding CPE behavior at high frequency with $\alpha = 0.834$ and ideal capacitive behavior at low frequency with $\alpha = 1$.

An alternative extension to low frequency is obtained by considering $\rho_0$ to be infinitely large and including a parallel path for current flow with resistance $R_p$. The value for $R_p$ was obtained from the characteristic frequency $f_0 = 170 \text{ Hz}$ using

$$R_p = \frac{\delta}{2 \pi \varepsilon_0 f_0}$$  \hspace{1cm} (20)

where, as was used above, $\delta = 31 \mu\text{m}$. The value of 56 k\Omega \cdot \text{cm}^2 obtained for $R_p$ is in good agreement with the value of $Z_f(0)$ obtained for the impedance expressed in terms of $\rho_0$. The resulting impedance response is shown as a solid line in Fig. 8. The impedance response in this case is symmetric in a Nyquist plot, yielding CPE behavior with $\alpha = 0.834$ at both high and low frequency. The parallel path for current flow may be considered to arise from transport through skin pores. It is evident that, while the model with a parallel current path shows better agreement with experimental data, neither model accounts fully for the complexity of skin behavior. This lack of agreement, however, does not influence the application of Eq. 12 for assessing skin thickness, since this interpretation requires only the high-frequency values.

Discussion

CPE behavior is commonly seen in the impedance response of electrochemical systems, and the determination of physical properties from interpretation of the impedance response remains a challenging problem. The two prevailing approaches in the literature are those of Brug et al. and Hsu and Mansfeld. The Brug formula was developed for surface distributions of capacitance and does not apply to the dielectric response of films. The Hsu–Mansfeld formula was developed for normal distributions, but Hirschorn et al. showed that the film thickness obtained using this formula can be too small.

The Hsu–Mansfeld formula was derived solely on the premise that, independent of the origin of the time-constant distribution, the time constant corresponding to the frequency for which the imaginary part of the impedance has a maximum magnitude can be expressed as $\tau_0 = R_C \varepsilon_0$. In contrast, the development by Hirschorn et al. identified a specific normal resistivity distribution that exhibits CPE behavior.

The resistivity values at the extremities of the film, $\rho_0$ and $\rho_\delta$, represent key parameters in the power-law model. CPE behavior is seen for frequencies that lie between the corresponding characteristic frequencies $f_0$ and $f_\delta$. For such data, neither $\rho_0$ nor $\rho_\delta$ can be determined unambiguously. The low-frequency behavior at frequencies below $f_0$ reflects ideal capacitive behavior for which $\alpha = 1$. The parameter $\rho_0$ can be determined unambiguously in this case. The high-frequency behavior for frequencies greater than $f_\delta$ also reflects ideal capacitive behavior for which $\alpha = 1$. The parameter $\rho_\delta$ can be determined unambiguously in this case.

While the power-law impedance may be applied over a complete range of frequency, a consequence of using a specific distribution is that physical properties can be inferred from the high-frequency portion of the spectrum, even for data that show CPE behavior over the entire measured high-frequency range. Graphical methods detailed by Orazem et al. can be used to obtain the CPE parameters $Q$ and $\alpha$. Often, the dielectric constant is known for specific film compositions. While insertion of $Q$, $\alpha$, and $\varepsilon$ in Eq. 9 provides only the
product $\delta \rho_0^{1-\alpha}$, this quantity is weakly dependent on $\rho_0$ for $\alpha$ close to unity. Thus, an estimate for film thickness can be obtained. The examples presented here show that the power-law model for resistivity distribution yields estimated values that are in good agreement with either measured or expected values for film thickness. The examples presented here also show situations for which the Hsu–Mansfeld formula either cannot be used (aluminum oxide) or yields incorrect values for film thickness (stainless steel and human skin).

The present approach is limited to systems for which the CPE behavior can be attributed to distributions of film properties in the direction normal to the electrode surface. Such systems, however, are commonly encountered. This work may apply to the study of systems that exhibit a distributed dielectric response such as oxides, organic coatings, and biological membranes.

Conclusions

Hirschorn et al.\textsuperscript{4} showed that under assumption that the dielectric constant is independent of position, a normal power-law distribution of local resistivity is consistent with the CPE. In the present work, an analytic expression was developed, based on the power-law resistivity distribution, that relates CPE parameters to the physical properties of a film. This expression yielded physical properties, such as film thickness and resistivity, that were in good agreement with expected or independently measured values for such diverse systems as aluminum oxides, oxides on stainless steel, and human skin.

For the power-law resistivity distribution developed in the present work, a CPE behavior was seen for measurements made between the characteristic frequencies $f_0$ and $f_b$. The power-law distribution of local resistivity was shown to yield ideal capacitive behavior at frequencies that are sufficiently large and sufficiently small, as can be expected for any model that accounts for a distributed film resistivity. A symmetric CPE response was obtained by adding a parallel resistive pathway.

References