Polymer Extrusion

Introduction
Extrusion is widely used in various industries. In this experiment, we investigate plastic extrusion by the screw extruder shown in Fig. 1. Such extruders are most common in plastic industry.

What Happens in an Extruder?
The screw in the extruder rotates and develops sufficient pressure to force material to go through a die and produce products with desired geometry.

Components of an Extruder
There are five main components in an extruder: screw, extruder drive, barrel, feed hopper, and die. The heart of the extruder is a helical screw which facilitates transport, heating, melting, and mixing of plastic. An extruder drive is an electric motor which supplies power to rotate the screw. The stability and quality of products is highly dependent on the design of the screw. The screw is housed in the extruder barrel which provides heating and cooling capabilities. The raw material is supplied from the feed hopper which is connected to the barrel by a feed throat. The feed hopper is designed to hold plastic.
pellets and ensure a steady flow of the pellets into the barrel. The die is located at the outlet of the extruder and determines the shape of the product. There exist multiple types of dies, including tubing dies, flat film dies, and blown film dies. In this lab, we use an annular tubing die shown in Fig. 2. Note that the size and shape of the extruded products are not exactly the same as the size and shape of the die due to draw down, cooling, swelling, and relaxation of the extruded polymer (see the Theory section below).

![Figure 2. Cross-sections of the annular tubing die.](image)

**Extrusion Lines**

In addition to an extruder, an extrusion process requires upstream and downstream equipment. Major equipment of an extrusion line is resin handling, drying system, extruder, post-shaping or calibrating device, cooling device, take-up device, and cutter or saw. The extrusion line in our lab includes extruder, take-off roller, and cooling water trough shown in Figure 3.
Figure 3. Extrusion line in the lab contains extruder, take-off roller and cooling trough with control variables rotating speed $\omega$ of the screw, take-off speed $v_t$ of the roller, and is the internal air pressure $p_i$.

**Objectives for the Extrusion Experiment**

1. Determine the material flow rates for different screw rotating speeds $\omega$.
2. Determine dependence of dimensions of the produced tubes on the screw rotating speed $\omega$, take-off speed $v_t$, and pressure drop $p_i$.
3. Compare the experimental results with the theoretical prediction.
Theory

Theoretical Background

In this lab we use an annular structured tubing die to produce tubes of different dimensions. When a polymer melt is extruded through an annular die and stretched under tension to a desired diameter, a hollow tube is produced. This process involves a die swell followed by a draw down (or stretching) of the polymer melt as illustrated in Figure 4. When polymer melt flows out of the die exit, a die swell occurs, i.e. the polymer melt expands in the radial direction due to residual stress in the melt. Die swell is a very complicated phenomenon because it depends on the strain history (i.e., memory effect) in the die as well as the rheological properties (both viscous and viscoelastic) of the melt. Thus, prediction of it is very difficult. The die swell, however, is restricted to a very short region near the die exit and an analytic progress can be made by simply neglecting the die swell region and focusing on the draw-down region.

Further simplifying assumptions for the current analysis are:

a) Isothermal flow;

b) Axisymmetric flow;

c) Length of the draw-down region (L) is much longer than the radius of the exit hole of the die;

d) Viscous force is dominant and the viscoelastic force, inertia, gravity, and surface tension are negligible;

e) Newtonian fluid.

Although polymer melts are non-Newtonian and the viscoelasticity effects are often important, the flow considered in this lab is weakly extensional and slow. Furthermore, the shear strain is also very weak throughout the entire draw-down region for this free surface problem. Thus, the Newtonian assumption is not too restrictive. Considering that the viscosity of polymer melts are quite high, assumption (d) is also not very restrictive. As the draw-down region is typically exposed to the air, the temperature may decrease as it flows down due to the cooling by air. However, the temperature variation may not be very large unless air is blown against the polymer melt for forced convective cooling.
Assumption (b) allows us to use cylindrical coordinates to describe the polymer flow. As indicated in Figure 4, \( r \) and \( z \) are coordinates in the radial and axial directions, respectively; \( R_o(z) \) and \( R_i(z) \) are the outer and inner radii of the tube; \( \mathbf{u} \) is the polymer flow velocity. The polymer velocities in the \( r \)- and \( z \)-directions are denoted as \( u \) and \( w \), respectively.

**Governing Equations**
Under the assumptions (a)-(e), the continuity equation and the \( r \)- and \( z \)-directional momentum equations for the flow in the draw-down region are
\[
\frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial w}{\partial z} = 0
\]  
(1)

\[
\frac{\partial p}{\partial r} = \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial (ru)}{\partial r} \right) + \frac{\partial^2 u}{\partial z^2} \right]
\]  
(2)

\[
\frac{\partial p}{\partial z} = \mu \left[ \frac{1}{r} \frac{\partial (rw)}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right]
\]  
(3)

Boundary conditions at the outer and inner surfaces of the tube are as follows:

At \( r = R_o(z) \),

\[
u - \left( \frac{\partial R_o}{\partial z} \right) w = 0
\]  
(4)

\[
2 \left( \frac{\partial R_o}{\partial z} \right) \left( \frac{\partial u}{\partial r} - \frac{\partial w}{\partial z} \right) + \left[ 1 - \left( \frac{\partial R_o}{\partial z} \right)^2 \right] \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right) = 0
\]  
(5)

\[
p = \frac{2}{1 + \left( \frac{\partial R_o}{\partial z} \right)^2} \left[ \frac{\partial u}{\partial r} - \left( \frac{\partial R_o}{\partial z} \right) \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right) + \left( \frac{\partial R_o}{\partial z} \right)^2 \left( \frac{\partial w}{\partial z} \right) \right]
\]  
(6)

At \( r = R_i(z) \),

\[
u - \left( \frac{\partial R_i}{\partial z} \right) w = 0
\]  
(7)

\[
2 \left( \frac{\partial R_i}{\partial z} \right) \left( \frac{\partial u}{\partial r} - \frac{\partial w}{\partial z} \right) + \left[ 1 - \left( \frac{\partial R_i}{\partial z} \right)^2 \right] \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right) = 0
\]  
(8)

\[
p - p_i = \frac{2}{1 + \left( \frac{\partial R_i}{\partial z} \right)^2} \left[ \frac{\partial u}{\partial r} - \left( \frac{\partial R_i}{\partial z} \right) \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right) + \left( \frac{\partial R_i}{\partial z} \right)^2 \left( \frac{\partial w}{\partial z} \right) \right]
\]  
(9)

Eqs. (4) and (7) are the kinematic conditions, Eqs. (5) and (8) are the tangential stress balances, and Eqs. (6) and (9) are the normal stress balances at the outer and the inner surfaces of the tube. In Eq. (9), \( p_i \) is the pressure inside the tube (i.e., the internal pressure) that can be controlled. That is, the inner (and, hence, the outer) radius can be controlled by varying the internal pressure when other conditions are fixed.

Boundary conditions in the axial direction are as follows:
At \( z = 0 \): \[ R_i = r_i, \quad R_o = r_o, \quad w = w_o \] (10)
At \( z = L \): \[ w = w_L \] (11)

Here, \( w_o \) is the average velocity in the axial direction that is determined once the output flow rate and the die geometry (i.e., \( r_i \) and \( r_o \)) are specified. The axial velocity at \( z = L \) (i.e., position at which the fiber is quenched or solidified instantaneously), \( w_L \), is also known as the take-up velocity (\( v_t \)) that is controlled by the take-up device.

Eqs. (1)-(3) with boundary (4)-(11) can be solved analytically assuming that the ratio \( r_o/L \) is small. For this, it is convenient to introduce the following dimensionless variables:

\[
\begin{align*}
& r^* = \frac{r}{r_o} \quad z^* = \frac{z}{L} \\
& u^* = \frac{u}{r_o w_o/L} \\
& w^* = \frac{w}{w_o} \\
& p^* = \frac{p}{\mu w_o/L} \\
& F^* = \frac{F}{\pi \mu r_o^2 w_o/L}
\end{align*}
\]

The superscript * indicates the dimensionless variables. In the following, we omit the superscript for brevity. The solution of the equations (1)-(3) in dimensionless form is:

\[
R_i(z) = \left[ \frac{r_i^2 (1-r_i^2)/w(z)}{\exp\left(\frac{p_i (1-w(z))}{\alpha w(z)}\right) - r_i^2} \right]^{1/2}
\]

(12)

\[
R_o(z) = \left[ R_i^2(z) + \frac{(1-r_i^2)}{w(z)} \right]^{1/2}
\]

(13)

\[
w(z) = \exp(\alpha z)
\]

(14)

\[
\alpha = \ln\left(\frac{w_L}{w_o}\right)
\]

(15)

Here, \( r_i \) and \( r_o \) are known as the die geometry factors (i.e., inner and outer diameters of exit of the die), \( w_o \) is output rate at die exit, and \( w_L \) is the take-off velocity.
Sample Calculations

Figures 5 through 7 provide the theoretical predictions given by the Eqs. (12)-(15) for the following conditions:

- the outer radius of the die, $r_o : 2.788 \text{ mm (7/32” diameter)}$
- the inner radius of the die, $r_i : 1.588 \text{ mm (1/8” diameter)}$
- draw span, $L : 50 \text{ cm}$
- viscosity, $\mu : 1000 \text{ Pa-s}$
- density, $\rho : 0.92 \text{ g/cm}^3$
- mass flow rate, $m : 50 \text{ g/min}$
- take-up speed, $w(L) : 40 \text{ cm/s}$
- internal pressure, $p_i : 0 \text{ –} 60 \text{ Pa}$

Figures 5-7 show variation of velocity and the tube radii with $z$. In these figures, axial velocity $w$ is normalized to $w_o$, $z$ is normalized to $L$, and the tube radii $R_i$ and $R_o$ are normalized to $r_o$.

Figure 5. Dimensionless axial velocity.
Figure 6. Dependence of the inner and outer radii of the tube on the internal pressure.

Figure 7. Dependence of the ratio of the inner and outer radii on the internal pressure.
Notation

- **Velocity vector** $u$ (m/s)
- $u_r$: directional component of the velocity vector $u$ (m/s)
- $w_z$: directional component of the velocity vector $u$ (m/s)
- **Viscosity of the polymer melt** $\mu$ (Pa·s)
- Output speed at die exit $w_o$ (m/s)
- Take-off speed $w_L$ (m/s)
- Outer tube radius at die exit $r_o$ (mm)
- Inner tube radius at die exit $r_i$ (mm)
- Outer tube radius $R_o$ (mm)
- Inner tube radius $R_i$ (mm)
- Internal (air) pressure $p_i$ (pa)
- Screw rotating speed $\omega$ (rpm)
- Take-off speed $v_i$ (m/s)
- Mass flow rate of the polymer $m$ (kg/min)
- Volumetric flow rate of the polymer $V$ (m$^3$/min)
- Density of the polymer $P$ (kg/m$^3$)
- Outer tube diameter $D_o$ (mm)
- Inner tube diameter $D_i$ (mm)
- Thickness of the tube walls $\theta$ (mm)

References