Summary

This guide provides essential information for determining the minimum velocity necessary to fluidize a bed of particles using a gas flow within a vertical column. The minimum fluidization velocity can be determined experimentally using the methods described here and the measurement compared to theories, also described here. The experiment and analysis can be performed on different particle shapes and sizes. Additional options include investigating the hysteresis in the minimum velocity of fluidization between cycles.

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1 Introduction

Fluidized beds are used widely in chemical processing industries for separations, rapid mass and heat transfer operations, and catalytic reactions. A typical fluidized bed is a cylindrical column that contains particles and through which fluid, either gaseous or liquid, flows. In the case of fluidized bed reactors, the particles would contain a catalyst, and for separations, the particles might be an absorbent or adsorbent. The velocity of the fluid is sufficiently high to suspend, or fluidize, the particles within the column, providing a large surface area for the fluid to contact, which is the chief advantage of fluidized beds. As shown in Figure 1, fluidized beds range in size from small laboratory-scale devices to very large industrial systems.

Regardless of whether the fluidized bed is used for a separation or reaction, a key goal is to operate the bed at a flow rate that optimizes the application. Accurate models would aid significantly, but modeling, even at a qualitative level, of the complex dynamics in fluidized beds continues to challenge engineers and scientists. The challenge arises from the necessity of considering both the solid and fluid phases and the interplay between them to form a complete picture for understanding the properties of fluidization.

Figure 2 assists in understanding the inherent challenge: multiple flow patterns within fluidized beds can be observed depending upon the velocity of the fluid. For sufficiently low rates of flow, fluid passes through the void space between particles without disturbing them. This case where the bed of particles remains in place is referred to as a “fixed bed”. At higher rates of flow, the drag forces acting on the particles can exceed the gravitational forces and lift particles. However, when the bed of particles expands, the drag forces drop as fluid velocity in the void spaces declines. The result is a highly dynamic state to which we refer as fluidization. Regimes of fluidization which can be easily identified from qualitative observations include bubbling and slugging patterns at relatively low flow rates and turbulent flow patterns at higher flow rates. At very high rates of fluid flow, the drag force can exceed the net gravitational forces on individual particles, even when the particles are widely separated. In this regime of pneumatic conveying, particles are carried through the container and must be reintroduced externally.

You will investigate some key parameters that govern fluidization, including the dependence of the minimum fluidization velocity on particle shape, size and density. This is the point at which the fixed bed of particles transitions to the particulate regime.

Figure 1: Photographs on the cover page indicate the range of sizes of fluidization equipment and some of the industrial applications of the operation: a) a large-scale fluidized bed used for bio-reactions [1], b) a smaller fluidized bed used as a dryer system in the pharmaceutical industry [2], and c) a laboratory-scale fluidized bed used for testing and development of new processes [3].
Figure 2: The regimes of fluidization as a function of the fluid velocity. At very low flow rates (left), the particles behave as a porous media, or fixed bed. After the gas velocity surpasses a critical value, the particles become fluidized. (Schematic is based upon a similar diagram appearing in Perry *et al.* [4].)
2 Minimum Velocity of Fluidization

The minimum velocity at which a bed of particles fluidizes is a crucial parameter needed for the design of any fluidization operation. The details of the minimum velocity depend upon a number of factors, including the shape, size, density, and polydispersity of the particles. The density, for example, directly alters the net gravitational force acting on the particle, and hence the minimum drag force, or velocity, needed to lift a particle. The shape alters not only the relationship between the drag force and velocity, but also the packing properties of the fixed bed and the associated void spaces and velocity of fluid through them.

To find the minimum fluidizing velocity, $U_{mf}$, experimental and theoretical approaches can be used. Methods for calculating the flow rate at which fluidization occurs are described first, as a review of fundamental ideas that govern the behavior of the bed of particles. Then, a procedure for estimating the minimum velocity from experimental measurements is described.

2.1 Calculating the Minimum Velocity

The incipient point at which the fluid, or gas, flow causes the bed of particles to expand and lift into the vertical column is marked by a conceptually simple balance. At $U_{mf}$, the hydrodynamic drag force on the particles $F_d$, due to the flow of gas through the packed bed of particles, matches (or just exceeds), the net gravitational forces $F_g$,

$$0 = F_g + F_d,$$

where the balance had been made in the direction of gravity.\(^1\) The calculation has many similarities to evaluating the terminal velocity of a single particle in a flow. Here however, the balance must be performed on the entire bed of particles as shown in Figure 3.

2.1.1 Gravitational Forces

The net gravitational forces on the bed of particles must consider the weight $W$ of the particles and the buoyancy forces $F_b$,

$$F_g = W - F_b = (\rho_p - \rho_f) g V_p,$$

where $\rho_p$ is the density of the particles, $\rho_f$ is the density of the fluid, $g$ is the gravitational acceleration constant, and $V_p$ is the total volume of particles within the fluidized bed. For a low density fluid, such as gas, the buoyancy force represents a small correction to the net gravitational force.

Since the drag forces (see Section 2.1.2) are generally written in terms of the bed voidage, expressing the gravity force in the same way proves convenient. If the weight and density of

\(^1\)Please note that all vector quantities and balances are restricted to the vertical direction within this document; positive values are parallel with gravity and negative values are antiparallel.
Figure 3: The bed of particles and the force balance. When the weight of the particles ($W$) exceeds the buoyancy forces ($F_b$) and the drag forces ($F_d$) due to the fluid velocity $U$, the particles remain fixed in place. The velocity $U$ is the minimum fluidization velocity if a small increase of velocity, $\delta U$, causes the bed to expand by a small amount $\delta H$ over its original height, $H$.

If the particles is known, then the particle volume can be calculated. Using the definition of bed voidage $\xi_m$, the volume of the particles can be written as

$$V_p = A H (1 - \xi_m), \quad (3)$$

where $A$ is the cross sectional area of the fluidized bed and $H$ is the height of the bed of the particles prior to the onset of fluidization. Note that Eq. 3 provides one method of calculating the bed voidage; a discussion of this, other methods of determining the bed voidage, and a discussion of particle packing is given in Appendix A.

### 2.1.2 Hydrodynamic Drag Forces

The local pressure drop through a porous medium is a function of the bed voidage, the flow velocity, and details of the particles,

$$\nabla P = f (\xi, U, D_e, \Phi_s), \quad (4)$$

and other properties of the fluid. The velocity $U$ is the superficial velocity, or volumetric flow rate of the fluid normalized by the cross-sectional area of the column. The equivalent volume diameter $D_e$ and sphericity factor $\Phi_s$ account for the details of the particle size and shape; for a spherical particle, the sphericity equals one and the equivalent diameter is simply the diameter of the sphere.²

For a homogeneous bed of monodisperse particles where $\xi$ is equal to $\xi_m$ everywhere, the pressure gradient $\nabla P$ can be integrated over the bed of particles to give the pressure drop

²Unless noted otherwise, the continuing discussion will assume that the bed is composed of spheres of diameter $D$. For non-spherical particles, replace the diameter $D$ by $\Phi_s D_e$ in all equations.
\[ \Delta P \text{ over the height } H \text{ of the bed, or } \Delta P = \int_0^H (\nabla P) \, dz = \nabla P \, H. \] The drag force on the bed of particles can then be calculated by multiplying by the cross-sectional area of the column,

\[ F_d = \nabla P A H. \] (5)

The pressure gradient, or drag force, depends on the flow velocity in a non-simplistic manner. However, different regimes of flow can be easily identified, much like the well-known case of the drag force on a single particle. The regimes are defined in terms of the Reynolds number,

\[ \text{Re}_p = \frac{D U \rho_f}{(1 - \xi_m) \mu}, \] (6)

where \( \mu \) is the viscosity of the fluid and the voidage enters the traditional definition to correct for the use of the superficial velocity \( U \). Figure 4 shows measurements of the packed bed friction factor,

\[ f_p = \frac{D \xi^3}{\rho_f U^2 (1 - \xi)} \nabla P, \] (7)

as a function of the Reynolds number. The relationship exhibited in Figure 4 holds only for the pressure drop prior to the incipient point of fluidization, when the particles are packed and the pattern of flow is that of a uniform, porous medium. For \( \text{Re}_p \leq 10 \), the flow is inertialess and the relationship between \( f_p \) and \( \text{Re}_p \) is linear. At very high values of the Reynolds number \( (\text{Re}_p \geq 1000) \), the flow is considered to be within the inviscid Newton region and \( f_p \) is independent of \( \text{Re}_p \).

In the viscous, or inertialess regime, the relationship between the pressure drop and flow
is linear. This is embodied within the commonly used Carman-Kozney equation [6; 7],

$$\nabla P = 180 \frac{\mu U (1 - \xi)^2}{D^2 \xi^3}. \quad (8)$$

This relationship is equivalent to writing $f_p = 180/\text{Re}_p$, which corresponds to the variables used in Figure 4. This equation is derived from an idealized model of a packed bed of particles. In the model, the tortuous path followed by the fluid passing through the particle bed is replaced by a set of parallel cylinders having the same flow resistance.

For the inviscid case, the Burke-Plummer equation applies [8],

$$f_p = 1.75; \quad (9)$$

this expression can be determined by a simple inspection of the data presented in Figure 4. Within this regime, the pressure drop is proportional to the square of the flow velocity.

To bridge the gap between the Carman-Kozney and Burke-Plummer equations, Ergun [5] proposed the correlation,

$$f_p = \frac{150}{\text{Re}_p} + 1.75, \quad (10)$$

which is a linear combination of the viscous and inviscid relations, albeit with a modified constant of 150 instead of 180 as given by Kozney [6] and Carman [7]. This is perhaps the most widely used equation for describing flow through porous media [9]. The reason is clear upon examining the solid line (Ergun’s correlation) in Figure 4, which fits the experimental data with fidelity over the entire range of Reynolds numbers.

### 2.1.3 Solving for $U_{mf}$

Balancing the forces as indicated in Equation 1, using Equations 2-3 for $F_g$ and equations given in Section 2.1.2 to determine $F_d$, results in expressions that can be solved to determine $U_{mf}$, the velocity $U$ at which fluidization occurs. All other quantities must be either measurable or known from another source; if the latter, be sure to explicitly indicate the source and the value used. Note that the error in the measurements and values from other sources propagate into an error for the calculation of the minimum velocity of fluidization. This error must be considered when assessing the comparison to the experimentally determined value of $U_{mf}$ as described in Section 2.2.

Rather than balancing the forces as in Equation 1, some methods of predicting $U_{mf}$ rely upon a direct correlation. One such result was developed by Wen & Yu [10],

$$\text{Re}_{mf} = \left(33.7^2 + 0.0408\text{Ar}\right)^{1/2} - 33.7, \quad (11)$$

where $\text{Ar}$ and $\text{Re}_{mf}$ are the Archimedes number and a modified Reynolds number. The Archimedes number is a ratio of gravity and viscous forces,

$$\text{Ar} = \frac{D^3 \rho_p (\rho_p - \rho_f) g}{\mu^2}, \quad (12)$$
Figure 5: Diagram of the pressure drop $\Delta P$ as a function of the superficial velocity $U$. The negative sign is indicative of the fact that the drag force acts in opposition to the velocity $U$. The point E marks the velocity $U_{mf}$ at which fluidization occurs.

where on must use $D_e$ instead of $D = \Phi_s D_e$. The modified Reynolds number is given by

$$Re_{mf} = \frac{DU_{mf} \rho_f}{\mu}. \quad (13)$$

Equation 11 can be solved directly to give a predicted value of $U_{mf}$ after the definitions of $Ar$ and $Re_{mf}$ are substituted into the expression.

### 2.2 Experimental Evaluation of $U_{mf}$

Measurements of the pressure drop across the bed of particles can be used to identify the minimum velocity of fluidization. As diagrammed in Figure 5, the pressure drop increases with flow rate until the bed expands and increases the porosity (point A). Note that the velocity and pressure drop relationship is not necessarily linear as shown, depending upon the range of $Re_P$ covered.

Upon further increasing the velocity, the pressure drop attains a maximum value. Between points A and B, the frictional drag force causes the particles to rearrange, which can alter the voidage. Upon rearrangement, the pressure decreases and point B lies above point C as a result. As $U$ is increased beyond point C, the pressure drop remains approximately constant until some point D where the velocity is not significantly greater than at point C. If the process is reversed by steadily lowering the velocity $U$, point E will be found instead of point B due to the different voidage resulting from the rearrangement of the particles, and line EF is the process for reforming the fixed bed of particles.

This conceptual diagram provides the basis for the experimental determination of $U_{mf}$. To identify point E, the fluid velocity is increased until the pressure goes through a maximum
and then ceases to change; this method defines the line CD. The rate of fluid flow is then reduced to get the line EF. The minimum fluidization velocity is the velocity at which these two lines intercept.

The increments in velocity must be small to resolve point E. Also, the curve represented in Figure 5 is a pseudo-steady one: after each increment in velocity $U$, sufficient time must be given for the pressure drop to equilibrate. Finally, the experimentally determined value of $U_{mf}$ is subject to error, which should be considered before comparing to any calculated values from Section 2.1.

3 Laboratory Objectives

Using the laboratory scale column that is described in Section 4, you will measure the minimum velocity of fluidization, using the ideas presented in Section 2.2. The measurements should be compared to predicted values to determine the efficacy of the various approximations made in Section 2.1. Aside from testing the case of spherical particles, you can

- investigate the minimum velocity of fluidization for cylindrical particles and particles that are flakes, and
- perform a study to examine the variation in the minimum fluidization velocity from cycle to cycle.

Limits on your time in the laboratory will prevent you from pursuing all of the possible objectives; keep in mind that thoroughness is preferred to completing all the possible tasks. Also, your instructors may ask you to perform some variations on the stated objectives.

4 Experimental Methods

The experimental apparatus shown in Figure 6 will be used in your investigation of the minimum velocity of fluidization. You will utilize only gas flow and can experiment with three types of particles: spherical particles, cylindrical particles, and flakes. In each case, the particles are monodisperse. The equipment is sufficiently instrumented to provide you with data that can be used to determine the minimum velocity of fluidization using the concepts presented in Section 2.2. Appendix B provides operating procedures for starting-up and shutting-down the equipment, as well as information about the particle sizes and equipment.

You should come to the laboratory prepared to run the equipment and perform your experiments, as time will be limited. To do so, you should be familiar with the operating procedures in Appendix B and you should have a plan of the steps you will need to take in order to meet the goals of the experiment. This includes a list of the parameters that will need to be set or measured during your time in the laboratory.

When you report on your experiment, note any deviations you make from the listed procedures and comment on any suggestions for improvements that you may have. Please, do not simply list the procedures in your report; rather reference this manual.
Figure 6: The experimental apparatus in the unit operations laboratory. The diagram shows the major elements of the experimental equipment and Table 1 contains the key for the various parts. The following components are not visible in this plot since they are behind the panel: supply tank for water, compressed air reservoir, pump for water, compressor for air.

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<td>Manometer for differential air pressure</td>
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Table 1: Key for the labels on the experimental equipment shown in Figure 6.
References


A Comments on Bed Voidage and Particle Packing

A key parameter that must be measured independently is the bed voidage, $\xi$. The bed voidage appears in the various equations used to predict the relationship between the drag force and fluid velocity. There are at least two methods available to you for estimating the bed voidage.

In the first method, you can rely directly upon Equation 8 which asserts that the pressure drop is proportional to the flow velocity. This process of determining $\xi$ requires performing a series of measurements of $\Delta P$ versus $U$ at low values where the relationship is linear. After verifying that the measurements are within the linear regime, the numerical value of the slope can be compared to $180\mu(1-\xi)^2/D^2\xi^3$ and, after some analysis, the void fraction can be estimated.

A second, and more direct, method of measuring $\xi$ is afforded by the definition of the bed voidage: the fraction of the total volume of the bed not occupied by the solid particles,

$$\xi = \frac{V_t - V_p}{V_t}. \quad (14)$$

The total volume of the bed ($V_t$) is given by the cross sectional area of the tube $^3$, $A$, and the height $H$ over which the particles are placed into the tube (see Figure 3). The volume of particles, $V_p$, is given by the mass $m$ of particles divided by their density, and

$$\xi = 1 - \frac{m}{\rho_p A H}. \quad (15)$$

Note that the two methods will likely predict slightly different values of the bed voidage, $\xi$. However, the two estimates should agree within the measurement errors, which must be propagated through the analysis. Also the value of $\xi$ can depend upon how the particles are loaded into the fluidization cell. For example, you may find that the value of $U_{mf}$ changes from one test to the next because the bed voidage has changed.

B Operating Procedures

The following text lists the operating procedures for the experiment. Please take care to follow all instructions and perform the experiments safely.

B.1 Start-up

1. Connect the power supply.
2. Connect the manometer to the experimental system using the provided connections. Secure all the hoses at the designated points.
3. Fully open the bypass valves for air.
4. Fully close the needle valve on the rotameter.
5. Start the compressor with the relevant switch and check that it is functioning properly.

\(^3\text{The tube diameter is 44 mm.}\)
B.2 Shut-down
1. Fully open the bypass valves for air.
2. Fully close the needle valves on the rotameter.
3. Turn-off the compressor.
4. Disconnect the power supply.
5. Recover the particles from the test vessel and clean the vessel.

B.3 Calibration of the pressure vs. flow
1. Increase the air flow rate in increments of 1 L/min by adjusting the needle valve and then the bypass valve.
2. Continuously note the air flow rate and corresponding differential pressure.
3. Continue the measurements up to the maximum flow and note the data.

B.4 Filling the test vessel with particles
1. Loosen the four knurled screws.
2. Lift the air filter off the cylinder flange, and set aside the air filter with the knurled screws and spacer sleeves.
3. Pour the mass into the cylinder, and measure the weight of particles before and after filling.
4. Place the spacer sleeves, air filter and knurled screws in their original position and tighten the knurled screws.
5. Read the initial bed height from the scale.

B.5 Fluidization and defluidization
1. Increase the air flow rate in 1 L/min increments by adjusting the needle valve and then the bypass valve.
2. Continuously note the air flow rate and corresponding differential pressure and bed height.
3. Continue the measurements up to the maximum flow and note the decrease data.
4. Repeat steps 1 through 3.

C Particle properties
Types of particles available for the experiment:

- Polystyrene spheres (density = 1.2 g/cm$^3$):
  - Diameter 0.4-0.6 mm
  - Diameter 0.6-1.0 mm
  - Diameter 1.0-2.0 mm
• Glass spheres (density = 2.4 g/cm$^3$):
  – Diameter 0.18-0.30 mm.

• Alumina spheres (density = 3.7 g/cm$^3$):
  – Diameter 0.5 mm
  – Diameter 0.8 mm
  – Diameter 1.0 mm

• Polyamide cylinders (density 1.1 g/cm$^3$)
  – Length 0.5 mm; diameter 0.5 mm.

• Plastic square flakes (density = 1.2 g/cm$^3$) of dimension 780 $\times$ 780 $\times$ 190 $\mu$m.