Assessment of Pipeline Condition Using Heterogeneous Input Data

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An inverse analysis model was developed which provides a mathematical framework for interpretation, in terms of the condition of a buried pipeline, of survey data in the presence of random noise. A boundary-element forward model was coupled with a weighted nonlinear regression algorithm to obtain pipe surface properties from two types of survey data: soil-surface potentials and local values of current flowing through the pipe. The model was demonstrated for synthetic data generated for a section of a coated underground pipeline electrically connected to a vertical sacrificial anode. The success of the regression was sensitive to the relative weighting applied in the objective function to the respective types of data. A generalized weighted and scaled objective function is proposed.


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In the past 10 years, significant progress has been made in the development of inverse models for cathodic protection problems. In 1995, Amaya et al. described use of a boundary element (BEM) solution of Laplace’s equation for a two-dimensional enclosure by which potentials measured at a distance from coupled electrodes could be used to estimate the galvanic corrosion rate. In 1997, Inglese constructed a regularized numerical method for a two-dimensional enclosure with a small aspect ratio. In a separate application, Aoki et al. used an inverse BEM analysis to eliminate ohmic error from measurement of polarization curves. In 1997, Aoki and Amaya described the coupling of a genetic algorithm with solution of Laplace’s equation for a two-dimensional enclosure to allow optimization of a cathodic protection system. The technique was extended to three-dimensional objects such as ship hulls, and numerical issues with the genetic algorithm were refined.

Miltiadou and Wrobel describe the application of genetic algorithms to a two-dimensional inverse problem in which potential data were used to assess surface characteristics. Wrobel and Miltiadou describe the application of genetic algorithms to three-dimensional inverse problems, including identification of coating holidays on buried pipelines. One important issue identified in their work was that to analyze large structures, potential sensors (e.g., reference electrodes) needed to be placed in close proximity to the defective region. Qiu and Orazem used a weighted nonlinear simulated-annealing regression algorithm in an inverse model for three-dimensional pipelines. The soil surface potentials were used as inputs to the model, and a sequential addition of parameters to the model allowed identification of the appropriate number of fitted parameters. An emphasis was placed on interpretation of survey data in the presence of random noise.

All the inverse models developed to date rely on a single type of data, i.e., potential readings from soil surface surveys or from individual sensor electrodes. As indicated by Wrobel and Miltiadou, for successful identification of coating defects, potential readings should be in close proximity to the coating defects. Even one meter of top cover may be sufficient to obscure the effects of localized underprotected regions. Thus, in some cases a serious coating defect may lead to insignificantly changes in potential at the soil surface. The low sensitivity of potential measurements to coating failures can be mitigated by including other types of data, for example, current densities in the pipeline metal. A schematic representation of the incorporation of potential survey and current data is given in Fig. 1.

Masilela and Pereira describe the use of soil surface potential gradient surveys to assess the condition of pipeline coatings. Fischer et al. discuss the difficulties of obtaining IR-free measurements of soil potential. Gummow and Eng describe use of four-electrode methods to measure currents in pipelines. Aerial surveys with magnetometers can be used to assess the flow of current in pipelines. Conversely, the presence of pipelines under cathodic protection conflicts interpretation of high-resolution aeromagnetic data. Campbell and Zimmerman describe the use of remote sensing of current in the Trans-Alaska Pipeline. Murphy et al. coupled impedance spectroscopy with use of superconducting quantum interference device (SQUID) magnetometer for remote sensing ofmodel developed by Qiu and Orazem. The inverse model comprises a BEM forward model which serves as part of the objective function for a nonlinear weighted simulated-annealing regression algorithm. The forward and inverse models are presented in the following sections.

The objective of this work was to develop a mathematical framework for interpretation of different types of survey data in the presence of random noise. Several key issues needed to be addressed in this work. Because field data is subject to both stochastic and systematic errors, the inverse strategy needed to account for the different error structures expected for different types of survey data, and the inverse strategy needed to address the different relative sensitivity of survey data types to pipe condition.

Mathematical Development

The inverse model developed here represents an extension of the model developed by Qiu and Orazem. The inverse model comprises a BEM forward model which serves as part of the objective function for a nonlinear weighted simulated-annealing regression algorithm. The forward and inverse models are presented in the following sections.

Forward model.—A simplified version of the forward model with assumptions for the pipe coating resistivity and varying pipe steel potential has been created from which the potential on soil surface and the current density along the pipe can be obtained. A preliminary version of the model has been presented by Qiu and Orazem.

CP system.—Following Qiu and Orazem, a schematic representation of the pipe-anode configuration modeled is presented in Fig. 2 in which a coated pipe is buried horizontally under the ground surface and connected by a wire to a vertical cylindrical anode. The underground region is considered to be bounded by the soil surface and to extend infinitely in the other directions. Laplace’s equation governs potential within the soil domain. The potential at infinity is obtained from the requirement that no current flows into or out of the domain.

Description of the pipe surface.—To reduce the number of parameters to be determined by the inverse model, Qiu and Orazem introduced a continuous function for coating resistivity as

\[ \rho = \rho_0 + \sum_k \rho_k e^{-(x-x_k)^2/(2\sigma_k^2)} \]

where \( \rho \) is the local coating resistivity, \( \rho_0 \) is the nominal resistivity...
of a defect-free coating, \( \rho_k \) is the reduction in coating resistivity associated with the defect \( k \), \( x_k \) is the center of the coating defect, and \( s_k \) represents the half-width of the defect region. Use of Eq. 1 has significant advantages over assigning a unique value of resistivity to each cylindrical element. For example, use of Eq. 1 to represent a single coating defect on a 1 km pipe discretized with a 1 m spacing would require only four parameters, as compared to 1000. The degree of freedom for the problem is increased dramatically, and the required computational time is thereby reduced.

**Steel potential.**—The model of Qiu and Orazem\(^9\) was extended to account for the electrical resistance of the steel pipe, which introduces an attenuation of the potential of the pipeline steel. The axial variation of potential in the steel can be significant for long pipelines or for pipelines in which a significant level of current is flowing. In addition, calculation of steel potential allows calculation of the current flowing in the pipe, which can be compared to field measurements.

The approach taken is similar to that described by Brichau and Deconinck\(^{18}\) and Aoki et al.\(^{19}\). The method of calculation is illustrated schematically in Fig. 3, where a horizontally placed pipeline is connected by a wire to a vertically placed sacrifice anode. The connection points are \( c_1 \) on the pipe and \( c_2 \) on the anode. In Fig. 3, \( i \) represents the current density along the axial direction of pipeline or anode, and subscripts \( p \) and \( a \) designate pipeline and anode, respectively. The current density entering the pipe at the ends is designated by \( i^0_p \) and \( i^n_p \), and the current density entering the anode at the ends is designated by \( i^0_a \) and \( i^n_a \). The current density entering the pipe coating in the radial direction is given by \( q \). As shown in Fig. 3, the protecting current flows away from the anode to the soil, then flows to the pipeline. On the pipeline, the currents flow from the two opposite directions to the connection point \( c_1 \). The total current \( I \) flows back to the anode through the wire.

The current density in the pipeline steel along the axial direction is given by

\[
i = \frac{1}{\rho_{steel}} ( - \nabla \cdot V ) = \frac{1}{\rho_{steel}} \frac{dV}{dz} \tag{2}\]

where \( V \) represents the potential of the steel in the pipe. Equation 2 can be integrated in the axial direction along a segment with length \( dz \), shown in Fig. 4, such that

\[
\int_{V_{k-1}}^{V_k} dV = - \rho_{steel} \int_{z_{k-1}}^{z_k} dz\]

\[
\bar{q} = \frac{A_s}{A_c} V^k - V^{k-1} \tag{3}\]
The potential change across the segment is given by

\[ V_k - V_{k-1} = -\rho_{steel}(z_k - z_{k-1}) \int_{z_{k-1}}^{z_k} \left( \frac{z - z_{k-1}}{z_k - z_{k-1}} \right) i^k dz \]

or

\[ V_k - V_{k-1} = \rho_{steel}(z_k - z_{k-1}) \left( \frac{1}{2} i_{k-1} + i_k \right) \]

where the potential difference between two points is found as the product of the average current density \((i_{k-1} + i_k)/2\) and the resistivity \(\rho_{steel}(z_k - z_{k-1})\).

Conservation of charge requires that the current flowing into the pipe segment is equal to the current flowing out. Thus

\[ i_{k-1} = i_k + \frac{A_p}{\bar{q}} \]

where \(\bar{q}\) is the average current density entering the coated surface of the element in the radial direction and \(A_p\) and \(A_s\) represent the areas of the steel cross section and sidewalks, respectively. For \(n\) pipeline segments, Eq. 5 and 6 comprise \(n\) equations in terms of \(n + 1\) unknown variables \(V_0, V_1, \ldots, V_n\). The remaining condition used specifies the reference potential \(V = 0\). In this study, the reference zero potential was set at the connection point \(c_1\) between the pipe and anode, i.e., \(V_{c_1} = 0\).

The current flows in the anode from the connection point \(c_2\) to the two different directions along the anode. The \(m\) equations, corresponding to \(m\) segments of anode, are given in terms of \(m + 1\) unknown variables \(V_p, V_1, \ldots, V_m\). The remaining condition needed to specify the potential was obtained as

\[ V_{c_2} = V_{c_1} - IR_{wire} = 0 \]

where \(R_{wire}\) is the resistance of the wire connecting the pipe to the anode and \(I\) is the current flowing through the wire.

The \(n + 1\) equations for the pipeline can be summarized as

\[ K_p V_p = T_p^{i_{ends}} + F_p i_p \]

and the \(m + 1\) equations for the anode can be summarized as

\[ K_a V_a = T_a^{i_{ends}} + F_a i_a \]

where \(K, T, and F\) are the coefficient matrices of vector \(V, i_{ends}\) and \(q\), respectively. Following the classic texts on the subject,\textsuperscript{20-23} the boundary element equations, including the concept of self-equilibrium, can be expressed in matrix form as

\[
\begin{bmatrix}
H_{pp} & H_{pa} & -1 \\
H_{ap} & H_{aa} & -1 \\
0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\phi_p \\
\phi_a \\
\phi_v
\end{bmatrix} = \begin{bmatrix}
G_{pp} & G_{pa} \\
G_{ap} & G_{aa} \\
A_p & A_a
\end{bmatrix} \begin{bmatrix}
(-n \cdot \nabla \phi)_p \\
(-n \cdot \nabla \phi)_a \\
\end{bmatrix} + \begin{bmatrix}
I_p \\
I_a
\end{bmatrix}
\]

Equation 8 and 9 can be combined with Eq. 10 to obtain a general matrix form for a set of equations as

\[
\begin{bmatrix}
K_p & 0 & 0 & 0 \\
0 & K_a & 0 & 0 \\
0 & 0 & H_{pp} & H_{pa} \\
0 & 0 & H_{ap} & H_{aa} \\
0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
V_p \\
\phi_p \\
V_a \\
\phi_a \\
\end{bmatrix} = \begin{bmatrix}
F_p & T_p & 0 & 0 \\
0 & 0 & F_a & T_a \\
G_{pp} & 0 & G_{pa} & 0 \\
G_{ap} & 0 & G_{aa} & 0 \\
A_p & A_{p,ends} & A_a & A_{a,ends}
\end{bmatrix} \begin{bmatrix}
(-n \cdot \nabla \phi)_p \\
(-n \cdot \nabla \phi)_a \\
\end{bmatrix}
\]

\[ [11] \]

Equation 11 represents a series of linear equations that can be solved simultaneously for the potential in the soil adjacent to the pipe and for the potential of the pipe and anode metal.

Analysis of the simulation results.—To explore the nature of the solution obtained by the forward model, a system was studied in which a 500 m pipe was buried 1.45 m (4.75 feet) below the soil surface. The pipe diameter was 0.457 m (1.5 feet). An anode was placed 50 m away from the center of the pipe. The diameter of the anode was 0.152 m, and its length was 1.22 m (4 ft). The soil resistivity was 100 \(\Omega\) m. The boundary conditions are shown in Fig. 2. A 0.5 mm-thick coating was assumed to cover the exterior surface of the pipe, with the exception of the two ends, which were assumed to be insulated. The pipeline was connected to the anode by a wire at the 50 and the 0.5 m position, respectively. The potential of the pipe steel was allowed to vary along the pipe. A coating defect was set on the middle of the length of pipeline. The forward model, Eq. 11, was used to calculate the potential on the soil surface and the current density along the pipeline.

The simulation results of pipeline are shown in Fig. 5. The coating resistivity of the pipeline is shown in Fig. 5a, where the nominal resistivity of defect-free coating is seen to be \(5.0 \times 10^7 \Omega\) m. Coating defects were modeled by a reduction of coating resistivity. The resistivity was assumed to decrease abruptly at a position of 250 m, corresponding to the position of a significant coating defect. The cathodic radial current density increased dramatically at the position of the coating defect, as shown in Fig. 5b.

The variation of potential within the pipe steel is presented in Fig. 5c. Because the length of the 500 m pipe was not large, the potential drop over the pipe was very small, on the order of 0.02 mV. The calculation of potential was primarily useful for allowing calculation of the current flow in the pipe. The steel potential was set to a value of zero at the 50 m position where the anode and pipe were connected. This represents the minimum value for steel potential. The current coming from soil enters the pipeline and flows along the pipe from two opposite directions to the wire connecting point on the pipe. Hence, the potentials on either side of the connection position have higher values. A change in the slope of the potential is seen at the position of the coating defect.

The axial current distribution shown in Fig. 5d reveals that the current density at the two ends of pipe was set to zero values. The current density changed sign at the bond location where the wire was connected to the pipe. A significant step increase in current is observed at the coating defect due to the contribution of the enhanced axial current density.

Inverse Model

To create the inverse model, the BEM code described in the previous section was used as part of the objective function in a nonlinear regression algorithm. The simulated annealing optimization approach was selected for the present work.\textsuperscript{9} Simulated annealing is suitable for large-scale problems and can search for a global minimum, which may be hidden among many local minima. Simulated annealing was found to be better than downhill simplex be-
cause the simplex method accepts only downhill steps during its searching, whereas simulated annealing can accept both downhill and uphill steps. In this way, simulated annealing can step out of the local minima and successfully locate the global minimum.\footnote{9}

Results

The inverse model acts to minimize the value of an objective function which represents the difference between measured and calculated values. The parameter set which results in the smallest value of the objective function can be assumed to reflect the condition of the coated pipe. The inverse results are strongly dependent on the form of the objective function chosen.

Regression to noise-free data.—Preliminary regression results were obtained using synthetic data generated with machine precision. The physical situation consisted of a single pipe connected to a single anode, as presented schematically in Fig. 2. The pipeline was 100 m long. Three coating defects were assumed to exist on the pipe coating with parameters presented in Table I. The forward model was used to obtain synthetic surface potential and current density data which were then used as the field data for the inverse model. The synthetic data are shown in Fig. 6. Dashed lines indicate the location of coating defects. The presence of defects at 30 and 70 m positions can be inferred from subtle changes in the current and surface potential distributions shown in Fig. 6; however, it is difficult to discern the defect located at 40 m.

The potential data set comprised 303 values corresponding to three lines of 101 points each located directly above the pipe segment and 1 m to either side. There were 99 current density data points. Regression was performed to homogeneous data sets which comprised either the current or potential data. Regression was also performed to heterogeneous data sets which included both current and potential data.

Regression to homogeneous data sets.—An objective function, which describes the difference between the measured potential and the calculated potential on the soil surface, or the difference between the measured current density along the pipe and the calculated current density data, is given as

$$g(x, \theta, \sigma) = \sum_{k=1}^{N} (i_k - \hat{i}_k)^2$$  \hspace{1cm} [12]

for current survey data and
Table I. Regression results using either Eq. 13 or 12 for homogeneous synthetic data without added noise or Eq. 14 for heterogeneous synthetic data without added noise. The initial values for each defect was \( x_0 = 50 \) m, \( \rho_k = -3.5 \times 10^7 \) \( \Omega \) m, and \( \sigma_k = 0.92 \) m.

<table>
<thead>
<tr>
<th>Data type</th>
<th>Equation</th>
<th>Coating defect</th>
<th>Position ( x_k (\text{m}) ) Set defect values</th>
<th>Resistivity ( \rho_k (\Omega \text{m}) )</th>
<th>Dimension ( \sigma_k (\text{m}) )</th>
<th>( \chi^2/\nu )</th>
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<tbody>
<tr>
<td>Current 12</td>
<td>1</td>
<td>30</td>
<td>(-4.5 \times 10^3)</td>
<td>0.316</td>
<td>6.7 \times 10^{-11}</td>
<td></td>
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<tr>
<td>2</td>
<td></td>
<td>40</td>
<td>(-3.0 \times 10^6)</td>
<td>0.548</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>70</td>
<td>(-2.0 \times 10^3)</td>
<td>0.447</td>
<td></td>
<td></td>
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<tr>
<td>Potential 13</td>
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<td>29.91</td>
<td>(-4.94 \times 10^3)</td>
<td>0.217</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>70.00</td>
<td>(-1.95 \times 10^3)</td>
<td>0.460</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Potential 13</td>
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<td>(-3.90 \times 10^3)</td>
<td>0.999</td>
<td>9.7 \times 10^{-4}</td>
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</tr>
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<td></td>
<td>30.03</td>
<td>(-2.62 \times 10^3)</td>
<td>2.37</td>
<td>1.2 \times 10^{-7}</td>
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<tr>
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<td>(-1.60 \times 10^3)</td>
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<td>8.0 \times 10^{-8}</td>
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<td>(-2.66 \times 10^3)</td>
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<td>(-0.50 \times 10^3)</td>
<td>2.46</td>
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\[
g(x, \rho, \sigma) = \sum_{j=1}^{N_p} (\psi_j - \hat{\psi}_j)^2 \quad [13]
\]

for potential survey data. The objective function value \( g \) is a function of the coating defect parameter vectors \( x, \rho \), and \( \sigma \), shown in Eq. 1. In Eq. 12 and 13, \( \dot{i}_k \) and \( \dot{\psi}_j \) represent the measured current density along the pipeline and soil surface potentials, and \( \dot{i}_k \) and \( \dot{\psi}_j \) represent the corresponding values obtained by the mathematical model. The number of current density data points is given by \( N_c \), and the number of potential data points is given by \( N_p \). Both types of data are influenced by the pipe coating condition; therefore, the minimum of the objective function is obtained when the coating parameter space reflects the condition of the pipeline coating.

An exploration of the regression strategy is presented in Table I, where regression was performed using either Eq. 13 or 12 for homogeneous synthetic data without added noise. The influence of initial guesses for the nonlinear regression was minimized by assigning equal initial parameter values for each defect, i.e., \( x_k = 50 \) m, \( \rho_k = -3.5 \times 10^7 \) \( \Omega \) m, and \( \sigma_k = 0.92 \) m.

The \( \chi^2/\nu \) statistic is presented in Fig. 7 as a function of the number of coating defects assumed in the regression. The method proposed by Qiu and Orazem\(^4\) for determination of the maximum number of defects revealed a clear minimum at two coating defects when regression was made to current data alone. One defect was located at the 30 m position and the other was at the 70 m position. Because the resistivity reduction of the set defect at the 40 m position was relatively small, it was difficult to find this defect by regression to current data alone.

The method for selecting the statistically significant coating defects yielded more equivocal results when regression was made to surface potential data only. No clear minimum could be found. When regression was made to a model with one coating defect, the defect found was that at the 30 m position. A small increase in the \( \chi^2/\nu \) statistic was found by regressing to two coating defects. The inverse results of applying potential and current density data to regression to two coating defects. The second coating discovered was located at the 43 m position, which was close to the defect set at the 40 m position. The smallest value of the \( \chi^2/\nu \) statistic was found, however, for three coating defects, but the results, showing two defects located at the same position, were not statistically significant. Use of the Akaike information criteria\(^24-26\) did not yield a clearer definition of the maximum number of resolvable parameters. The difficulty of obtaining the statistically significant number of coating defects for regression to potential data indicates that work is needed to establish refined methods for evaluating the regression results.

Better inverse results were obtained by using current density data, even though there were fewer current data than potential data.

**Unweighted regression to heterogeneous data sets.**—In order to take advantage of the information from both the potential and the current density data, an objective function was obtained as the summation of the least-squares differences of the two sets of data, i.e.

\[
g(x, \rho, \sigma) = \sum_{j=1}^{N_p} (\psi_j - \hat{\psi}_j)^2 + \sum_{k=1}^{N_c} (i_k - \hat{i}_k)^2 \quad [14]
\]

The inverse results of applying potential and current density data to Eq. 14 are shown in Table I. As shown in Fig. 7, the \( \chi^2/\nu \) statistic indicated that two defects could be identified, and these were located near the 30 and 70 m positions. The results obtained using Eq. 14 are comparable to those obtained using the current data alone. Use of Eq. 14 for regression to heterogeneous data did not improve the resolution of pipe coating condition.
Weighted regression to heterogeneous data sets.—The regression strategy used for Eq. 14 failed because the magnitudes of the two types of data were vastly different. A weighting strategy was employed in which each contribution to the regression was scaled by the magnitude of the corresponding model values, i.e.,

$$g(x, p, \sigma) = \sum_{j=1}^{N_p} \frac{(\psi_j - \hat{\psi}_j)^2}{\hat{\psi}_j^2} + \sum_{k=1}^{N_c} \frac{(i_k - \hat{i}_k)^2}{\hat{i}_k^2}$$  \hspace{1cm} [15]

Under the assumption that the weighting strategy should be based on the variance of the data,27,28 use of Eq. 15 is consistent with the assumption of a proportional error structure.

The $\chi^2/v$ statistic is presented in Fig. 8 as a function of the number of coating defects assumed in the regression. Each of the three coating defects could be found. The holiday 1 approximately has the same coating parameters as those of the defect at the 30 m position. The position of holiday 2 is close to the defect at the 40 m position, with a relative error for position of about 3%. The holiday 3 refers to the defect at the 70 m position. For the holidays 2 and 3, the coating resistivity reduction and the defect width differ from the set values. This could be attributed partially to the correlation among defect parameters evident in Eq. 1.

The importance of weighting for regression strategies is particularly evident for heterogeneous data sets. Proportional weighting is generally recommended for synthetic data for which the uncertainty is governed by the accuracy of the calculation. For experimental data, a weighting strategy should be guided by the perceived or measured variance of the experimental data.27,28

**Regression to noisy data.**—For the purpose of testing the robustness of the inverse model, noise was added to the data set. Noise was added to the potential data according to

$$\psi = \psi^* + \sigma_\psi \cdot \mathbf{P}(0, 1)$$  \hspace{1cm} [16]

where $\mathbf{P}(0, 1)$ represents a normally distributed random number with mean value equal to zero and standard deviation equal to unity. The randomly generated number was scaled by a standard deviation, which was assigned a value $\sigma_\psi = 0.4 \text{ mV}$. The noise in the current signal was assumed to be proportional to the current such that

$$i = i^* [1 + \alpha_i \cdot \mathbf{P}(0, 1)]$$  \hspace{1cm} [17]

The value of $\alpha_i$ was 0.02; thus, the noise level for the current measurement represented 2% of the measured value. The amount of noise associated with current measurements depends on the techniques employed. While the level of current noise assumed may be appropriate for four-electrode current measurements, the noise associated with magnetometer readings is much larger than the 2% assumed here.

The comparison between the base value and the noise-added value for the potential data and the current density data is shown in Fig. 9a and b, respectively. While the objective function (Eq. 15) was well suited for regression to noise-free synthetic data, only the largest coating defect at the 30 m position could be found when it was used to regress to the synthetic data with added noise. The situation was not improved by replacing the proportional weighting shown in Eq. 15 with weighting based on the assumed error structure.

**Weighted and scaled regression to heterogeneous data sets.**—The reason for the poor results obtained by regression of Eq. 15 could be related to the large number of potential measurements which are relatively insensitive to coating condition. A rescaled objective function, similar to that employed by Aoki et al.5 to remove ohmic potential drop from polarization curves, was proposed as

$$g(x, p, \sigma) = \frac{1}{N_p} \sum_{j=1}^{N_p} \frac{(\psi_j - \hat{\psi}_j)^2}{\sigma_\psi^2} + \frac{1}{N_c} \sum_{k=1}^{N_c} \frac{(i_k - \hat{i}_k)^2}{\sigma_i^2}$$  \hspace{1cm} [18]

where $\sigma_\psi^2$ and $(\sigma_i)^2 = (\alpha_i i_k)^2$ provide a weighting based on the error structure of the data. Equation 18 provides an objective function in which the contributions of potential and current data are scaled such that they provide equal weight to the regression.

The regression results are shown in Table II. The parameter values obtained when two defects were assumed present are in good agreement with input values for the defect located at the 30m and 70m locations. The parameter values obtained when three defects were assumed were inconsistent with the input defect conditions. Unreliable estimates were typically obtained when overfitting data. The weighted $\chi^2/v$ statistic was used to identify the optimal number of defects for the regression. The weighted $\chi^2/v$ statistic reached a minimum value for two coating defects, indicating that only two defects could be identified.

The scaled objective function (Eq. 18) differs from that used by Aoki et al.5 in two important respects: the current used corresponds to the current flowing along the length of the pipe rather than the local value normal to the pipe surface, and the weighting is based on
the error structure of each type of data rather than on the range of data values. Both objective functions serve to reduce the influence of superfluous input information.

Generalization of the objective function.—The proper scaling of the objective function is determined by the relative sensitivity of a particular type of data to the coating condition. A form of Eq. 18 that is readily generalizable to multiple types of data can be expressed as

\[ g_k(x_r, r_s) = \sum_{k=1}^{M} \lambda_k g_k \]

[19]

where \( g_k \) is the objective function for a given type of field data scaled by the number of data, i.e.

\[ g_k(x_r, r_s) = \frac{1}{N_k} \sum_{j=1}^{N_k} \frac{(x_{kj} - \hat{x}_{kj})^2}{(\sigma_{kj})^2} \]

[20]

and \( \lambda_k \) is a scaling factor that accounts for the relative sensitivity of the data type to coating condition. To facilitate interpretation of the \( \chi^2/\nu \) statistic, it is appropriate to constrain \( \lambda_k \) such that

\[ \sum_{k=1}^{M} \lambda_k = 1 \]

[21]

In Eq. 20, \( x_{kj} \) and \( \hat{x}_{kj} \) represent the measured and calculated values, respectively, for data type k, and \( \sigma_{kj} \) represents the corresponding standard deviation for the measured values. The data sets may comprise, among others, surface on-potential, surface off-potential, current density data, or readings from buried reference electrodes or coupons.

Conclusions

The inverse analysis model developed here provides a mathematical framework for interpretation of survey data which contains random noise in terms of the condition of a buried pipeline. Within this model, a boundary element forward model was coupled with a weighted nonlinear regression algorithm to obtain pipe surface properties from two types of survey data: measured soil-surface potentials and local values of current flowing through the pipe. The technique identified the location of coating anomalies as well as the breadth of the anomaly and the amount that the local resistivity had changed. An algorithm was developed that could be used to identify the maximum number of coating anomalies that can be detected. This number was found to be sensitive to the quality of data as well as to the actual coating condition. If the number of coating anomalies detected is smaller than the actual number of coating defects, the technique identifies the most serious anomalies.

Table II. Regression results using Eq. 18 for heterogeneous synthetic data with added noise. The initial values for each defect were \( x_k = 50 \) m, \( \rho_k = -3.5 \times 10^7 \) \( \Omega \) m, and \( \sigma_k = 0.92 \) m.

<table>
<thead>
<tr>
<th>Data type</th>
<th>Equation</th>
<th>Coating defect</th>
<th>Position ( x_k ) (m)</th>
<th>Resistivity ( \rho_k ) (( \Omega ) m)</th>
<th>Dimension ( \sigma_k ) (m)</th>
<th>( \chi^2/\nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both</td>
<td>17</td>
<td>1</td>
<td>30</td>
<td>(-4.5 \times 10^7)</td>
<td>0.316</td>
<td>3.02</td>
</tr>
<tr>
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<td>2</td>
<td>40</td>
<td>(-3.0 \times 10^7)</td>
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<td>2.71</td>
</tr>
<tr>
<td>Both</td>
<td>17</td>
<td>3</td>
<td>70</td>
<td>(-2.0 \times 10^7)</td>
<td>0.447</td>
<td>1.50</td>
</tr>
<tr>
<td>Regression result</td>
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<td>30.1</td>
<td>(-4.59 \times 10^7)</td>
<td>0.50</td>
<td>3.02</td>
</tr>
<tr>
<td>Both</td>
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<td>30.1</td>
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<td>0.31</td>
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<tr>
<td>Both</td>
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<td>69.4</td>
<td>(-0.77 \times 10^7)</td>
<td>1.27</td>
<td>10.8</td>
</tr>
<tr>
<td>Both</td>
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<td>3</td>
<td>38.8</td>
<td>(-2.05 \times 10^7)</td>
<td>0.88</td>
<td>10.8</td>
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<tr>
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<td>46.4</td>
<td>(-0.66 \times 10^7)</td>
<td>2.07</td>
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<tr>
<td>Both</td>
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<td>46.8</td>
<td>(-2.13 \times 10^7)</td>
<td>1.50</td>
<td>1.50</td>
</tr>
</tbody>
</table>
The model was demonstrated for synthetic data generated for a section of a coated underground pipeline electrically connected to a vertical sacrificial anode. The success of the regression was sensitive to the relative weighting applied in the objective function to the respective types of data. For a given data type, the weighting was based on the stochastic noise level of the data. To date, the model has been applied only to synthetic data, but this work demonstrates the feasibility of coupling a boundary element forward model with a nonlinear regression algorithm to obtain pipe surface properties from pipeline survey data.

References